

Worksheet 26.1 - Special Relativity – Chapter 26 (Do these on your own paper pls.)

Objective C: Time and mass dilation, length contraction:

Moving clocks run slowly, gain mass, and shrink in the dimension parallel to their velocity:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \Delta t = \gamma \Delta t_o \quad L = \frac{L_o}{\gamma} \quad \text{Not in the data packet: } m = \gamma m_o$$

Problems for:

1. A clock flies by us at 0.780c. How much time will it take that clock to register 60.0 seconds? (95.9 s)
2. A clock flies by us at 0.780c. When a clock in our reference frame has registered 60.0 seconds, how much time has registered on the moving clock? (37.5 s)
3. A moving clock takes 61.0 s to register 60.0 s. How fast is it moving? (0.180 c)
4. A bus is 45.0 feet long at rest. If it is going by at 0.450c, what is its length as we observe it? What is its length as observed by people on the bus? If the bus goes through a tunnel that has a proper length of 45 feet, what length do the people in the bus observe it to be? Why is there not a single answer to the question “Does the bus fit inside the tunnel?” (40.2 feet, 45.0 feet, 40.2 feet, depends which reference frame)
5. A car going 0.470c has a length of 14.0 feet. What is its length at rest? (15.9 feet)
6. What speed does a 45 foot long bus need to go to fit exactly into a tunnel that is 40. feet long? (0.46c)
7. What is the mass of a 0.142 kg baseball that is pitched by Optimus Prime at 2.89×10^8 m/s? By how much does the mass increase? Where does this mass come from? (0.529 kg, 0.387 kg, from the KE)
8. A particle going 0.740c has a mass of 2.49×10^{-27} kg. What is its rest mass? (1.67×10^{-27} kg)
9. An electron has a rest mass of 0.511 MeV, and a moving mass of 1.511 MeV. (This means it has 1.00 MeV of kinetic energy – it has been accelerated through 1.00 Million volts.) What is its speed ? (0.941c)
10. Particles that take 1.56×10^{-6} seconds to decay on the average at rest, are sped up so the average lifetime is now 2.31×10^{-6} seconds. What is the speed of the particles? How far do they travel before decaying? (0.738c, 511 m)
11. Easy from Chapter 26: 1(42.6 m), 2(2.07×10^{-6} s), 3(1.00, 0.9998, 0.980, 0.312, 0.199, 0.0447), 4(69.1 Ly)
12. Moderate from Chapter 26: 5(0.773c), 6(0.90c), 8(0.436c)
13. Challenging from Chapter 26: 7(26 y), 10(0.141c), 11(2.7 y, 9.2 y), 12(11.0 y, 3.09 y, 2.97 y, 0.96c), 13(6.39 m, 1.25 m, 15.0 s, 0.660c, 15.0 s), **14(0.887c)**

Objective I: Relativistic addition of velocities

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} \quad u' = \frac{u + v}{1 + \frac{uv}{c^2}}$$

When you would subtract:

When you would add (Not in data packet):

Questions:

14. Why can't you just add relativistic velocities like you do low velocities?

Problems:

15. Rob the hamster rides to the right on a cart going 0.360 c. He throws a baseball at 0.680 c relative to him to the right. What is the velocity of the baseball in the earth frame? (0.835 c right)
16. Rob rides to the right on a cart going 0.490c. He throws a baseball. We observe the baseball going 0.980c to the right relative to the earth frame. With what velocity did Rob throw the ball in his frame? (0.943c right)
17. Rob the hamster is riding a flatbed rail car to the right at 0.870c, and throws a baseball to the left at 0.560c. What is the velocity of the ball in the earth frame of reference? (0.605c to the right)
18. Rob rides to the right on a cart going 0.360 c. He throws a baseball at 0.720c to the left. What is the velocity of the ball in the earth frame? (-0.486c (left))
19. Goldy the goldfish is riding in a van going to the right at 0.820c, and throws a baseball so that we see it going 0.350c to the right in our frame of reference. What velocity did Goldy give the ball in his frame of reference? (0.659c to the left)
20. Rob rides to the right on a cart going 0.710c. He shines a torch in the direction he is going. How fast do we see the photons from the torch moving in the earth frame. What if he shines it backwards? (c, -c)
21. Problems from chapter 26: 43(0.80c), 44(0.80c, -0.80c), 45(0.98c, -0.42c), 46(0.65c), 47(0.92c), 48(0.70c)

Name_____

Worst joke Murray ever made:_____

Show your work, circle your answers, and use sig figs to receive full credit.

1. What is the kinetic energy of a 0.142 kg baseball going $0.780c$?
2. What is the velocity of a 0.128 MeV electron?
3. A black hole has an event horizon of 4.30×10^6 m in radius. What must be its mass?
4. A very strong concertmaster is on top of a 148 m tall tower near a black hole where the gravitational field strength is 5.53×10^{12} N/kg. If another player is making a frequency of 440.0 Hz at the bottom, what frequency does the concertmaster hear at the top? What beat frequency do they hear?
5. A black hole has a mass of 21.0 solar masses. (The mass of the sun is 1.99×10^{30} kg.) *Calculate the radius of the event horizon, *Calculate the time it would take a clock 160. km beyond the event horizon to register 60.0 seconds of elapsed time as we observe it from a great distance.

Worksheet 26.2 - Energy and General Relativity (Do these on your own paper pls.)

Objective H: Relativistic Kinetic Energy

Kinetic Energy: $E_k = (\gamma - 1)m_0c^2$ **Rest Energy:** $E_o = m_0c^2$ **Total or Moving energy:** $E = \gamma m_0c^2$

Proton: $m_0 = 1.673 \times 10^{-27}$ kg = 938 MeV, **Electron:** $m_0 = 9.110 \times 10^{-31}$ kg = 0.511 MeV,

Questions:

- Where does the mass come from when it dilates (increases) with velocity.

Problems:

- A 1.2 kg object (rest mass) is moving at 2.85×10^8 m/s. What is its new mass, and what is its kinetic energy? (3.84 kg, 2.4×10^{17} J)
- What velocity must an electron have when it is accelerated through 340. kV? $m_0 = 9.11 \times 10^{-31}$ kg = 0.511 MeV (0.800c)
- A 0.00612 kg (rest mass) bullet is going so fast it has a (dilated) mass of 0.00645 kg. What is its kinetic energy, and what is its velocity (2.97x10¹³ J, 0.32c)
- The LHC will accelerate protons to about 7 TeV. If a proton has a rest mass of 938 MeV, what is the velocity of the protons in the LHC? (0.999999991c)
- A 0.146 kg baseball has a kinetic energy of 3.30×10^{16} J. What is its moving mass in kg? What is its velocity? (0.513 kg, 0.959c)
- An electron is going 0.980 c. What is its kinetic energy in MeV? (2.06 MeV)
- A 0.160 kg baseball is going so fast that it has a mass of 0.190 kg. How fast is it going? What is its kinetic energy in Joules? (0.539c, 2.70×10^{15} J)
- A proton is accelerated through 210. million volts. What is its velocity? (0.577c)
- A particle going 0.670c has a dilated mass of 147 MeV. What is its rest mass in MeV? (109 MeV)
- An object going 0.850 c has a kinetic energy of 6.20×10^{14} J. What is its rest mass in kg? (0.00767 kg)
- A particle has a rest mass of 410. MeV. How fast is it going if it has a kinetic energy of 110. MeV? (0.615c)
- A 0.0850 kg ball has a 8.60×10^{15} J of kinetic energy. How fast is it going? (0.882c)
- Through what potential (in volts) do you accelerated a proton so that it is going 0.850 c? (843×10^6 V, 8.43×10^8 V)
- What is the kinetic energy of a 0.170 kg baseball going 0.820c? (answer in Joules) (1.14×10^{16} J)
- What is the velocity of a 1.20 GeV (1GeV = 1000 MeV) Proton? (0.899c)
- Problems from chapter 26: 24(9×10^2 kg), 26(0.866c), 28(2.23×10^{-9} J, 6.46×10^{-18} kg m/s), 29(1.51×10^{-10} J, 8.70×10^{-19} kg m/s), 31(0.437c), 32(0.941c), 33(0.30c), 37(5.5×10^{19} J), 35(0.866c, 0.745c)

Schwarzschild Radius: $R_s = \frac{2GM}{c^2}$ (**R_s = radius of the event horizon - the point from which the $V_{esc} > c$**)

- What is the Schwarzschild radius of a 0.145 kg baseball? (2.15×10^{-28} m)
- What is the mass of a black hole with a radius of 1.00 m? (6.75×10^{26} kg)
- What is the Schwarzschild radius of a black hole with a mass of 21 million suns? ($M_{sun} = 1.99 \times 10^{30}$ kg) (6.19×10^{10} m)
- What is the mass of a black hole that has an event horizon with a radius of 1.50×10^{11} m? (1.01×10^{38} kg)

Frequency Shift: $\frac{\Delta f}{f} = \frac{g\Delta h}{c^2}$ (**Low clocks run slowly**)

- A very strong concertmaster is playing 440.00 Hz at the top of an 4.50 m tall tower on a neutron star where the “g” is 1.816×10^{14} N/kg. We are at the bottom also playing 440.00 Hz. What is the beat frequency we hear? Do we hear the player on the top of the tower as sharp (higher frequency) or flat? What frequency do we observe? (4.00 Hz, Sharp, 444 Hz)
- If we are living on a neutron star, and we tune the local station “Neutrock 91.7 (MHz) in at 90.2 on our FM Dial. We know that we are at a different elevation by 35.6 m. What is the “g” here? Are we higher or lower than the broadcast antenna of “Neutrock”? (4.13×10^{13} N/kg, Higher)
- A 417 nm spectral line is shifted to 423 nm through a distance of 1 A.U. What is the change in frequency? What is the “g” in the vicinity of source? (1.02×10^{13} Hz, 8510 N/kg)

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$$

Gravitational Time Dilation: (**Low clocks run slowly**) (**Low = closer to the event horizon**)

- A black hole has a Schwarzschild radius of 39 km. What time does it take a clock 44 km from the event horizon to register 6.0 hours as we observe it from a distance? (8.2 hours)
- A clock takes 173 minutes to register 120. minutes as we see it from a distance. It is 78.0 km from a black hole. What is the Schwarzschild radius of this black hole? What is its mass? (40.5 km, 2.73×10^{31} kg)
- A star orbiting 89 km from a black hole has a 656 nm line spectral that has gravitationally red-shifted to 712 nm. What is the mass of the black hole? (Use $v = f\lambda$ and $f = 1/T$ to find the periods of 656 and 712 nm) (9.1×10^{30} kg)