Worksheet 26.2 - Energy and General Relativity

Objective H: Relativistic Kinetic Energy

Kinetic Energy: $E_k = (\gamma - 1)m_o c^2$ **Rest Energy:** $E_o = m_o c^2$ **Total or Moving energy:** $E = \gamma m_o c^2$

Proton: $m_o = 1.673 \times 10^{-27}$ kg = 938 MeV, **Electron:** $m_o = 9.110 \times 10^{-31}$ kg = 0.511 MeV,

Questions:

1. Where does the mass come from when it dilates (increases) with velocity.

Problems:

2. A 1.2 kg object (rest mass) is moving at 2.85×10^8 m/s. What is its new mass, and what is its kinetic energy? (3.84 kg, 2.4×10^{17} J)
3. What velocity must an electron have when it is accelerated through 340. kV? $m_o = 9.11 \times 10^{-31}$ kg = 0.511 MeV (0.800c)
4. A 0.00612 kg (rest mass) bullet is going so fast it has a (dilated) mass of 0.00645 kg. What is its kinetic energy, and what is its velocity (2.97×10^{13} J, 0.32c)
5. The LHC will accelerate protons to about 7 TeV. If a proton has a rest mass of 938 MeV, what is the velocity of the protons in the LHC? (0.999999991c)
6. A 0.146 kg baseball has a kinetic energy of 3.30×10^{16} J. What is its moving mass in kg? What is its velocity? (0.513 kg, 0.959c)
7. An electron is going 0.980 c. What is its kinetic energy in MeV? (2.06 MeV)
8. A 0.160 kg baseball is going so fast that it has a mass of 0.190 kg. How fast is it going? What is its kinetic energy in Joules? (0.539c, 2.70×10^{15} J)
9. A proton is accelerated though 210. million volts. What is its velocity? (0.577c)
10. A particle going 0.670c has a dilated mass of 147 MeV. What is its rest mass in MeV? (109 MeV)
11. An object going 0.850 c has a kinetic energy of 6.20×10^{14} J. What is its rest mass in kg? (0.00767 kg)
12. A particle has a rest mass of 410. MeV. How fast is it going if it has a kinetic energy of 110. MeV? (0.615c)
13. A 0.0850 kg ball has a 8.60×10^{15} J of kinetic energy. How fast is it going? (0.882c)
14. Through what potential (in volts) do you accelerated a proton so that it is going 0.850 c? (843×10^6 V, 8.43×10^8 V)
15. What is the kinetic energy of a 0.170 kg baseball going 0.820c? (answer in Joules) (1.14×10^{16} J)
16. What is the velocity of a 1.20 GeV (1GeV = 1000 MeV) Proton? (0.899c)
17. Problems from chapter 26: 24(9×10^2 kg), 26(0.866c), 28(2.23×10^{-9} J, 6.46×10^{-18} kg m/s), 29(1.51×10^{-10} J, 8.70×10^{-19} kg m/s), 31(0.437c), 32(0.941c), 33(0.30c), 37(5.5×10^{19} J), 35(0.866c, 0.745c)

Schwarzschild Radius: $R_s = \frac{2GM}{c^2}$ (R_s = radius of the event horizon - the point from which the $V_{esc} > c$)

18. What is the Schwarzschild radius of a 0.145 kg baseball? (2.15×10^{-28} m)
19. What is the mass of a black hole with a radius of 1.00 m? (6.75×10^{26} kg)
20. What is the Schwarzschild radius of a black hole with a mass of 21 million suns? (Msun = 1.99×10^{30} kg) (6.19×10^{10} m)
21. What is the mass of a black hole that has an event horizon with a radius of 1.50×10^{11} m? (1.01×10^{38} kg)

Frequency Shift: $\frac{\Delta f}{f} = \frac{g \Delta h}{c^2}$ (Low clocks run slowly)

22. A very strong concertmaster is playing 440.00 Hz at the top of an 4.50 m tall tower on a neutron star where the "g" is 1.816 x 10^{14} N/kg. We are at the bottom also playing 440.00 Hz. What is the beat frequency we hear? Do we hear the player on the top of the tower as sharp (higher frequency) or flat? What frequency do we observe? (4.00 Hz, Sharp, 444 Hz)
23. If we are living on a neutron star, and we tune the local station "Neutrock 91.7 (MHz) in at 90.2 on our FM Dial. We know that we are at a different elevation by 35.6 m. What is the "g" here? Are we higher or lower than the broadcast antenna of "Neutrock" (4.13×10^{13} N/kg, Higher)
24. A 417 nm spectral line is shifted to 423 nm through a distance of 1 A.U. What is the change in frequency? What is the "g" in the vicinity of source? (1.02×10^{13} Hz, 8510 N/kg)

Gravitational Time Dilation: $\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$ (Low clocks run slowly) (Low = closer to the event horizon)

25. A black hole has a Schwarzschild radius of 39 km. What time does it take a clock 44 km from the event horizon to register 6.0 hours as we observe it from a distance? (8.2 hours)
26. A clock takes 173 minutes to register 120. minutes as we see it from a distance. It is 78.0 km from a black hole. What is the Schwarzschild radius of this black hole? What is its mass? (40.5 km, 2.73×10^{31} kg)
27. A star orbiting 89 km from a black hole has a 656 nm line spectral that has gravitationally red-shifted to 712 nm. What is the mass of the black hole? (Use $v = f\lambda$ and $f = 1/T$ to find the periods of 656 and 712 nm) (9.1×10^{30} kg)