

# IB Physics

## Chapter 8 Syllabus

### Angular Mechanics

| A/B   | In Class:   | Due on this class:  |
|---|---|---|
| 1<br>Feb<br>11/12                             | DI-Introduction/Demos<br>GW-Kinematics  | <b>VF 8ABC, 8D, 8E</b><br>Turn in: Angular Quantities 8.0 |
| 2<br>Feb<br>13/14                             | <b>SA8.1-Angular Kinematics (first 30)</b><br>VF-8F-Torque, 8G-Moment of Inertia<br>DI-Torque and Moment of Inertia Demos | Turn in: FA8.1  |
| 3<br>Feb<br>15/19                             | GW-8.2 Angular Dynamics<br>GW-FA8.2   |   |
| 4<br>Feb<br>21/22                             | <b>SA8.2-Angular Dynamics (first 30)</b><br>VF-8I Rolling Dynamics<br>DI-Rolling Dynamics Demo/Example                    | Turn in: FA8.2  |
| 4 <sup>3</sup> / <sub>4</sub><br>Feb<br>25/26 | Στυπιδ ασσ Oaks Testing   | <b>VF 8J, 8K 8L</b>                                       |
| 5<br>Feb<br>27/28                             | GW-Energy and Momentum 8.3<br>DI-Demos  | <b>VF 8M, 8NOP</b>  |
| 6<br>Mar<br>1/4                               | <b>SA8.3-Energy and Momentum (first 30)</b><br>VF-8Q, 8R<br>DI-Cross Product/Precession                                   | Turn in: FA8.3  |
| 7<br>Mar<br>5/6                               | DI-Gyroscopes<br>GW-Gyroscope Investigation   | <b>VF 8Q, 8R</b>  |
| Mar<br>7/8                                    | <b>Statics!!!!</b>  | <b>VF 9A, 9B, 9C</b><br>Turn in: Gyroscope Investigation  |



























1 Lab:

- Gyroscope Investigation

3 Formative/Summative Assessments

- 8.1 – Rotational Kinematics
- 8.2 – Rotational Dynamics
- 8.3 – Rotational Energy and Momentum

Handouts:

-  \_SyllabusAngular\_Mechanics2019
-  8
-  FA08.1
-  FA08.2
-  FA08.3
-  FAReviewInClass
-  Lab-GyroscopeInvestigation
-  Noteguide-02AngularDynamics
-  Noteguide-03AngularRollingAndEnergy
-  Noteguide-04AngularMomentum
-  Noteguide08ABC-BasicAngularQuantitiesAndConversions
-  Noteguide08E-AngularKinematics
-  Noteguide08F-Torque
-  Noteguide08G-MomentOfInertia
-  Noteguide08H-AngularDynamics
-  Noteguide08I-Rolling Dynamics
-  Noteguide08J-ComplexDynamics
-  Noteguide08K-RotationalKineticEnergy
-  Noteguide08L-RollingCOE
-  Noteguide08M-ComplexCOE
-  Noteguide08NOP-AngularMomentum
-  Noteguide08Q-VectorCrossProductandPrecession
-  Worksheet-AngularDynamics8.2
-  Worksheet-AngularEnergyAndMomentum8.3
-  Worksheet-AngularKinematics8.1
-  Worksheet-AngularQuantities8.0



## Angular Kinematics problems from 8.1

Tangential Relationships:  $s = \theta r$ ,  $v = \omega r$ ,  $a = \alpha r$

1. A 0.0760 m diameter (76 mm) skateboard wheel rolls through 137 rotations. What linear distance did it travel? (32.7 m)
2. What is the angular acceleration of a 0.630 m diameter bicycle wheel if it is accelerating linearly at 8.20 m/s/s? (26.0 rad/s/s)
3. A 0.0660 m diameter skateboard wheel travels 12.0 m. How many rotations does it go through? (57.9 rotations)
4. A 0.650 m diameter wheel accelerates at 1.54 rad/s/s. What is the tangential acceleration of the edge? (0.5005 m/s/s)
5. A wheel goes through 143 rotations when it rolls linearly 14.2 m. What is the radius of the wheel? (0.0158 m)

Tangential Relationships with unit conversions: **1 rev or rot =  $2\pi$  radians, 1 minute = 60 seconds**

6. What is the linear velocity 0.120 m from the center of a grinding disk spinning at 1450 RPM? (18.2 m/s)
7. What is the angular velocity of a 0.920 m radius aircraft tire in rotations/second when it has a linear velocity of 48.0 m/s? (8.30 rot/s)
8. A merry go round spins at 0.590 rotations/second. What is the tangential velocity 1.80 m from the center? (6.67 m/s)
9. A 0.940 m diameter wheel has a tangential velocity at its edge of 25.0 m/s. What is its angular velocity in RPM? (508 RPM)
10. A hard drive spins at 7200 RPM. What distance from the center has a tangential velocity of 12.0 m/s? (0.0159 m)

Simple Rotational kinematics:  $v = u + at$ ,  $s = (u+v)t/2$ ,  $v^2 = u^2 + 2as$ ,  $s = ut + \frac{1}{2}at^2$

11. A drill going 98.0 rad/s decelerates at -1.20 rad/s/s for 15.0 s. What is the final angular velocity in rad/s? (80.0 rad/s)
12. A drill speeds up from rest to 156 rad/s in 5.70 s. Through what angle in radians does it go? (445 rad)
13. A drill goes through 132 radians in 8.80 s slowing to rest. What was its initial angular velocity in rad/s? (30.0 rad/s)
14. A drill speeds up from 11.0 rad/s to 35.0 rad/s in 184 radians. What is its angular acceleration? (3.00 rad/s/s)
15. A drill goes through 526 radians accelerating at 2.58 rad/s/s from rest. What is its final angular velocity in rad/s? (52.1 rad/s)

Rotational Kinematics with unit conversions:

16. A motor speeds up from 1350. RPM with an angular acceleration of 2.90 rad/s/s for 19.0 seconds. Through what angle in radians does it rotate? (3210 rad)
17. A car tire initially rotating at 37.0 rotations per second slows down through 148 rotations in 5.20 seconds. What is its final angular velocity in rotations per second? (19.9 rot/s)
18. A drill speeds up from 680. RPM to 1540 RPM with an acceleration of 1.80 rad/s/s. How many rotations does it go through? (926 rotations)
19. A skateboard wheel speeds up from 5.30 rotations/sec to 12.0 rotations/s in 9.00 seconds. What is the angular acceleration in rad/s/s? (4.68 rad/s/s)
20. A turntable accelerates at 0.835 rad/s/s from rest to 33.3 RPM. What is its angular displacement in radians? (7.28 rad)

Rotational Kinematics with tangential relationships:

21. A car with 0.340 m radius tires going 19.2 m/s decelerates at 1.20 m/s/s for 2.30 s. What is the final angular velocity of the tires? (48.4 rad/s)
22. A car with 0.840 m diameter wheels accelerates from rest with an acceleration of 6.40 m/s/s for 3.50 seconds. Through what angle in radians do the wheels go? (93.3 radians)
23. A 0.110 m radius ball going 5.80 m/s rolls to a stop in 9.70 seconds. What was the angular acceleration of the ball in rad/s/s? (-5.44 rad/s/s)
24. A 0.360 m radius car tire goes from 12.5 rad/s to 36.8 rad/s with a linear acceleration of 3.90 m/s/s. What linear distance does the car travel? (19.9 m)
25. A 0.125 m radius grinding wheel speeds up from 142 rad/s to 259 rad/s in 13.0 s. Through what distance does a point in the edge of the wheel travel in this time? (326 m)

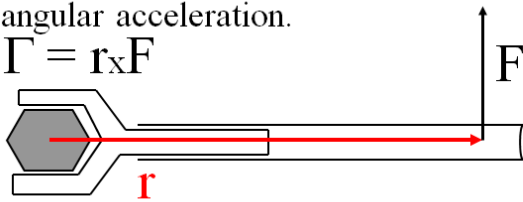


# Noteguide for Torque (Videos 8F)

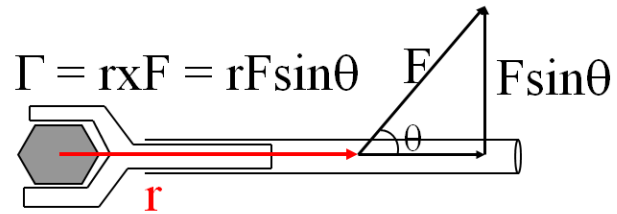
Name \_\_\_\_\_

Torque A twisting force that can cause an angular acceleration.

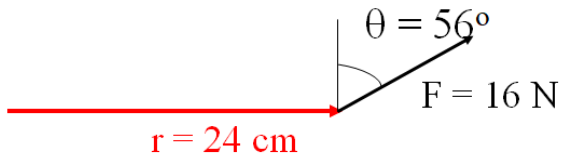
$$\Gamma = r \times F$$



If  $r = 0.50$  m, and  $F = 80$  N,  $\Gamma =$



Example: What's the torque here?



Whiteboards:

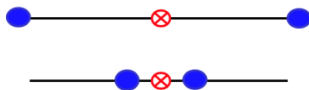
|  |   |
|--|---|
| <p>1. What is the torque when you have 25 N of force perpendicular 75 cm from the center of rotation? (19 mN)</p>  | <p>2. If you want 52.0 mN of torque, what force must you exert at an angle of <math>65.0^\circ</math> to the end of a 0.340 m long wrench?</p>  |
| <p>3. A. What force acting at <math>32.0^\circ</math> with a line perpendicular to the end of a 23.0 cm long wrench will generate 28.0 mN of torque about the left side of the wrench? (143.6 N)</p> <p><math>r = 23.0</math> cm</p> | <p>4. C. A 45.0 cm wrench makes a <math>29.0^\circ</math> angle above the horizontal. What is the torque about the left side of the wrench if a 48.0 N force is exerted vertically upward at the end? (18.9 mN)</p> |



**Noteguide for Moment Of Inertia (Videos 8G)**

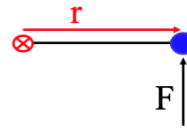
Name \_\_\_\_\_

Moment of Inertia - Inertial resistance to angular acceleration.



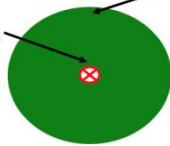
Question - If the blue masses were identical, would both systems respond identically to the same torque applied at the center?

$F = ma$  - We can't just use "m" for "I"  
 $\Gamma = I\alpha$  (The position of "m" matters!)



What about a cylinder rotating about its central axis?

Some parts are close to the axis



Some parts are far from the axis

In this case,  
 $I = \frac{1}{2}mr^2$

(You need calculus to derive it)

Three main ones:

$\frac{1}{2}mr^2$  - Cylinder (solid)

$mr^2$  - Hoop (or point mass)

$\frac{2}{5}mr^2$  - Sphere (solid)

Example: Three 40. kg children are sitting 1.2 m from the center of a merry-go-round that is a uniform cylinder with a mass of 240 kg and a radius of 1.5 m. What is its total moment of inertia?

(Whiteboards on the back)

Whiteboards;

1. What is the moment of inertia of a 3.5 kg point mass that is 45 cm from the center of rotation?  
(0.71 kg m<sup>2</sup>)

2. A uniform cylinder has a radius of 1.125 m and a moment of inertia of 572.3 kg m<sup>2</sup>. What is its mass?  
(904.4 kg)

3. A sphere has a mass of 45.2 grams, and a moment of inertia of  $5.537 \times 10^{-6}$  kg m<sup>2</sup>. What is its radius?  
(0.0175 m)



## Noteguide for Angular Dynamics (Videos 8H)

Name \_\_\_\_\_

The angular equivalent of  $F = ma$  is:

$$\vec{F} = m\vec{a}$$

$$\Gamma = I\alpha$$

Example: A string with a tension of 2.1 N is wrapped around a 5.2 kg uniform cylinder with a radius of 12 cm. What is the angular acceleration of the cylinder? How many rotations will it make before it reaches a speed of 2300 RPM from rest?

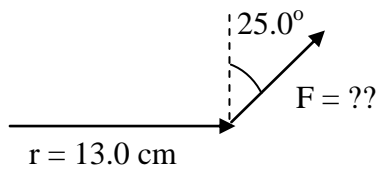
(Whiteboards on the back)

Whiteboards:

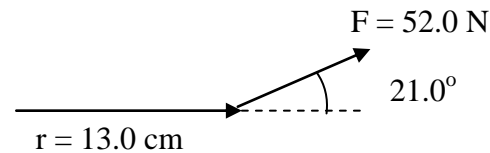
|   |  |
|---|--|
| <p>1. What torque is needed to accelerate a <math>23.8 \text{ kg m}^2</math> wheel at a rate of <math>388 \text{ rad/s/s}</math>?<br/>(9230 mN)</p>   | <p>2. An object has an angular acceleration of <math>23.1 \text{ rad/s/s}</math> when you apply <math>6.34 \text{ mN}</math> of torque. What is the object's moment of inertia?<br/>(<math>0.274 \text{ kgm}^2</math>)</p>   |
| <p>3. If a drill exerts <math>2.5 \text{ mN}</math> of torque on a <math>0.075 \text{ m}</math> radius, <math>1.75 \text{ kg}</math> grinding disk, what is the resulting angular acceleration?<br/>(<math>510 \text{ rad/s/s}</math>)</p>                | <p>4. What torque would accelerate an object with a moment of inertia of <math>9.3 \text{ kg m}^2</math> from <math>2.3 \text{ rad/s}</math> to <math>7.8 \text{ rad/s}</math> in <math>0.12 \text{ seconds}</math>? (1 hint)<br/>(430 mN)</p>   |
| <p>5. If you exert <math>12.0 \text{ N}</math> tangentially at the edge of a <math>45.0 \text{ kg}</math> <math>72.0 \text{ cm}</math> diameter cylindrical potter's wheel, what is its angular acceleration?<br/>(<math>1.48 \text{ rad/s/s}</math>)</p> | <p>6. A merry go round is a uniform solid cylinder of radius <math>2.0 \text{ m}</math>. You exert <math>30. \text{ N}</math> of force on it tangentially for <math>5.0 \text{ s}</math> and it speeds up from rest to <math>12.9 \text{ RPMs}</math>. What's its mass?<br/>(110 kg)</p> |

### Angular Dynamics problems from 8.2

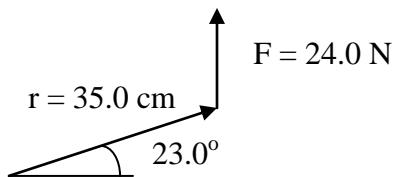
A. What force acting at  $25.0^\circ$  with a line perpendicular to the end of a 13.0 cm long wrench will generate 7.80 mN of torque about the left side of the wrench? (66.2 N)



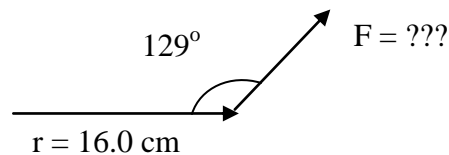
B. Calculate the torque about the left side of the wrench if 52.0 N acts at an  $21.0^\circ$  angle with the end of a 13.0 cm long wrench. (2.42 mN)



C. A 35.0 cm wrench makes a  $23.0^\circ$  angle above the horizontal. What is the torque about the left side of the wrench if a 24.0 N force is exerted vertically upward at the end? (7.73 mN)



D. A force is exerted at an angle of  $129^\circ$  with a 16.0 cm wrench as shown below. Calculate the force needed to create 3.80 mN of torque about the left side of the wrench. (30.6 N)



Moments of inertia: Cylinder:  $I = \frac{1}{2} mr^2$ , Sphere:  $I = \frac{2}{5} mr^2$ , Thin Ring or Point Mass:  $I = mr^2$

Simple  $F = ma$  problems:  $\Gamma = I\alpha$

1. A baton requires 5.70 mN of torque to accelerate at 18.4 rad/s/s about its center. What is the moment of inertia? (0.310 kgm<sup>2</sup>)
2. A flywheel with a moment of inertia of 0.859 kg m<sup>2</sup> accelerates at 13.0 rad/s/s. What is the torque? (11.2 mN)
3. A motor with 43.0 mN of torque accelerates at 153 rad/s/s. What is its moment of inertia? (0.281 kgm<sup>2</sup>)
4. A torque of 21.0 mN acts on a motor with a moment of inertia of 1.53 kg m<sup>2</sup>. What is the angular acceleration? (13.7 rad/s/s)
5. What torque will accelerate a motor with a moment of inertia of 3.87 kg m<sup>2</sup> at 6.60 rad/s/s? (25.5 mN)

$F = ma$  problems, but  $I = \frac{1}{2} mr^2$  (cylinder),  $\frac{2}{5} mr^2$  (sphere), or kinematics, or  $\Gamma = rF$

6. A 0.400 m diameter, 4.30 kg sphere accelerates about its center at 6.80 rad/s/s. What is the torque? (0.468 mN)
7. A drill with a moment of inertia of 0.0180 kg m<sup>2</sup> is slowed by a frictional torque of 0.270 mN. If it is moving at 142 rad/s, how many radians will it go through before it stops? (672 rad)
8. A grinding wheel with a diameter of 0.640 m and a moment of inertia of 0.172 kg m<sup>2</sup> decelerates at -8.90 rad/s/s because of a tangential friction force applied at the edge. What is this force? (4.78 N)
9. A torque of 19.0 mN acts on a flywheel with a moment of inertia of 3.20 kg m<sup>2</sup>. If it starts at rest, in what time will it go through 16.0 radians? (2.32 s)
10. A torque of 3.50 mN acts on a 7.10 kg, 0.132 m diameter shot put. (a sphere) What is the angular acceleration of the sphere? (283 rad/s/s)

Same as above with unit conversions:

11. A 0.219 m diameter bowling ball has a tangential force 5.50 N acting on it and it accelerates from rest going through 13.0 rotations in 3.21 seconds. What is the moment of inertia of the ball? (0.0380 kgm<sup>2</sup>)
12. A 0.310 m radius flywheel (essentially a thin ring) with a mass of 3.20 kg. What is its rate of deceleration if you exert a force of 2.20 N tangentially at its edge? (2.22 rad/s/s)
13. A flywheel is a 13.2 kg 1.80 m diameter thin ring. If you exert a force of 51.0 N tangentially at its edge, what is its angular acceleration? (4.29 rad/s/s)
14. A flywheel that is a 0.730 m diameter thin ring with a mass of 16.0 kg would require what torque to accelerate from rest to 1120 RPM in 8.10 seconds? (30.9 mN)
15. What is the moment of inertia of a 0.258 m radius flywheel if when you exert a tangential force of 11.5 N at the edge it accelerates from rest to 680. RPMs in 123 rotations? (0.904 kgm<sup>2</sup>)

Same as above with unit conversions and kinematics:

16. A 161 kg 4.72 m diameter (cylindrical) merry go round is sped up from rest by a 25.0 N force applied tangentially at its edge. What is its speed in RPMs after 38.0 seconds? (47.8 RPM)
17. A 2.10 m radius, 351 kg (cylindrical) merry go round spinning at 75.0 RPM slows to a halt in 11.5 rotations. What force applied tangentially at the edge would cause this? (157 N)
18. A 232 kg 4.10 m diameter (cylindrical) Merry go round is stopped from a speed of 94.0 RPM in 55.0 seconds. What frictional force applied tangentially at the edge would cause this? (42.6 N)
19. A 243 kg 1.70 m radius (cylindrical) merry go round stops from a speed of 68.0 RPM because of a frictional force applied at the edge of 8.50 N. How many rotations does it go through in stopping? (98.1 rotations)
20. A 4.60 m diameter (cylindrical) merry go round speeds up from rest going through 5.10 rotations in 41.0 seconds because of a 15.0 N force applied tangentially at the edge. What is the mass of the merry go round? (342 kg)

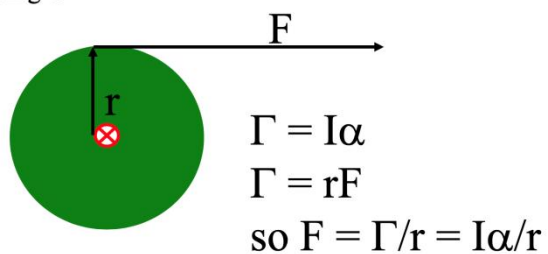


## Noteguide for Rolling Dynamics (Videos 8I)

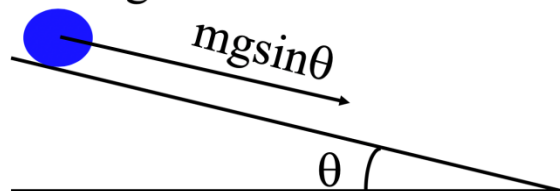
Name \_\_\_\_\_

Rolling objects accelerate linearly and angularly:

Force causing  $\alpha$



Rolling:



$$I = \frac{1}{2}mr^2$$

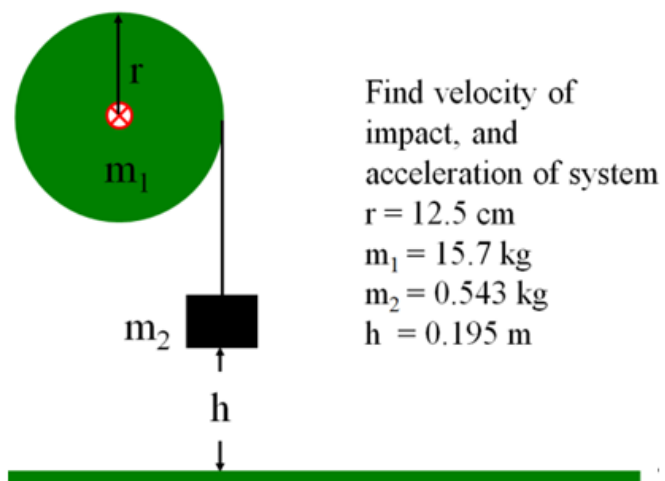
$$F = \Gamma/r = I\alpha/r$$

An 11.0 g, 0.0130 m radius cylinder rolls down an incline that is 2.90 m long, and loses 0.340 m of elevation. What is its acceleration down the plane, and its velocity at the bottom of the plane?

(Try the whiteboard on the back for a different object rolling down the incline)

A marble (a solid sphere:  $I = \frac{2}{5}mr^2$ ) has a mass of 23.5 g, a radius of 1.2 cm, and rolls 2.75 m down an incline that loses 0.650 m of elevation.

1. Solve for  $a$  in terms of  $g$  and  $\sin\theta$  ( $\frac{5}{7}g\sin\theta$ )
2. Plug in and get the acceleration (1.66 m/s/s)
3. suvat for the final velocity (3.02 m/s)



Find velocity of impact, and acceleration of system

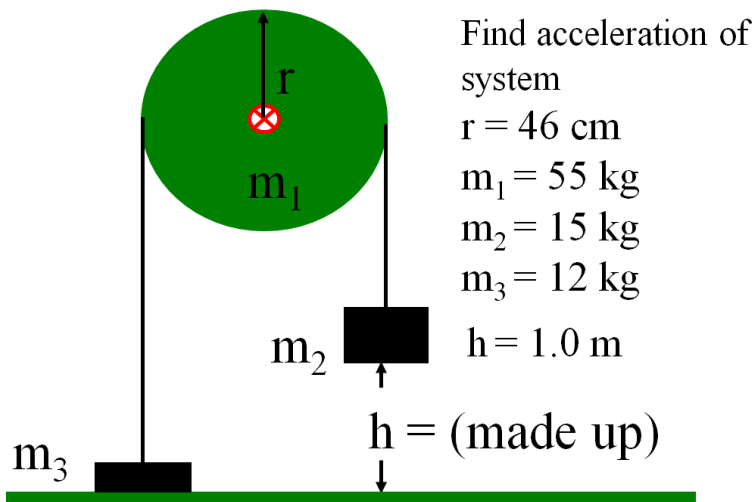
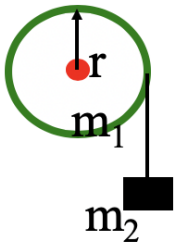
$$r = 12.5 \text{ cm}$$

$$m_1 = 15.7 \text{ kg}$$

$$m_2 = 0.543 \text{ kg}$$

$$h = 0.195 \text{ m}$$

A string is wrapped around a 12.0 cm radius 4.52 kg thin ring. A mass of 0.162 kg is hanging from the end of the string. What is the acceleration of the system, and what is the velocity of  $m_2$  when it has fallen 1.00 m? (Assume it is released from rest)





## Noteguide for Rotational Kinetic Energy (Videos 8K)

Name \_\_\_\_\_

Translational:  $E_{\text{kin}} = \frac{1}{2}mv^2$

Rotational:  $E_{\text{k rot}} = \frac{1}{2}I\omega^2$

Work:

$$W = Fs$$

$$W = \Gamma\theta$$

Example: A 23.7 kg 45 cm radius cylinder is rolling at 13.5 m/s at the bottom of a hill.  
What is its translational kinetic energy?

What is its rotational kinetic energy?

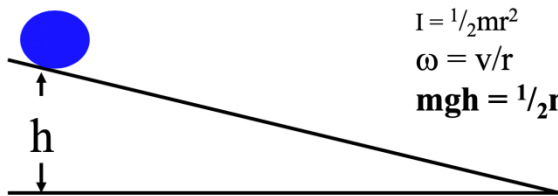
What is the total kinetic energy?      What was the height of the hill?

### Whiteboards

|   |  |
|---|--|
| <p>1. What is the rotational kinetic energy of an object with an angular velocity of 12.0 rad/s, and a moment of inertia of 56.0 kg m<sup>2</sup>?<br/>(4.0 x 10<sup>3</sup> J)</p> | <p>2. What must be the angular velocity of a flywheel that is a 22.4 kg, 54 cm radius cylinder to store 10,000. J of energy?<br/>(78 rad/s)</p>  |
| <p>3. What is the total kinetic energy (Translational and rotational) of a 2.5 cm diameter 405 g sphere rolling at 3.5 m/s?<br/>(3.5 J)</p>   | <p>4. If you exert 14.0 mN of torque through 3.10 rotations on a potter's wheel that is a 26.0 kg, 68.0cm diameter uniform cylinder, what will be the final angular velocity?<br/>(19.1 rad/s)</p> |

## Noteguide for Rolling COE (Videos 8L)

Name \_\_\_\_\_



$$I = \frac{1}{2}mr^2$$

$$\omega = v/r$$

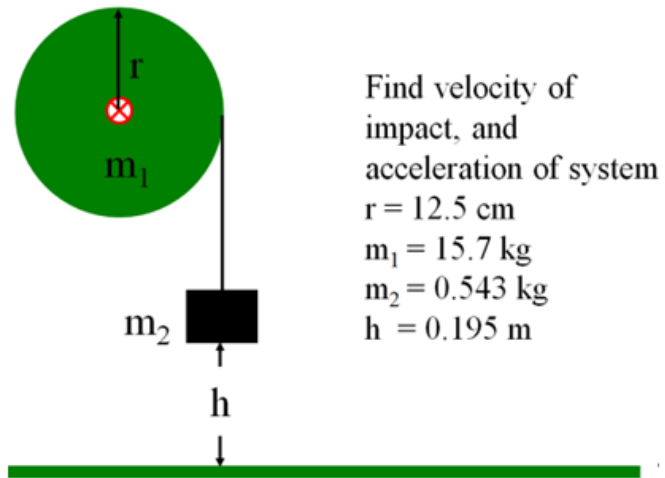
$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

An 11.0 g, 0.0130 m radius cylinder rolls down an incline that is 2.90 m long, and loses 0.340 m of elevation. What is its acceleration down the plane, and its velocity at the bottom of the plane?

### Try this one:

A marble (a solid sphere) has a mass of 23.5 g, a radius of 1.20 cm, and rolls 2.75 m down an incline that loses 0.650 m of elevation.

$$v = \sqrt{\frac{10}{7}gh}, 3.02 \text{ m/s}, 1.66 \text{ m/s/s}$$



Find velocity of impact, and acceleration of system

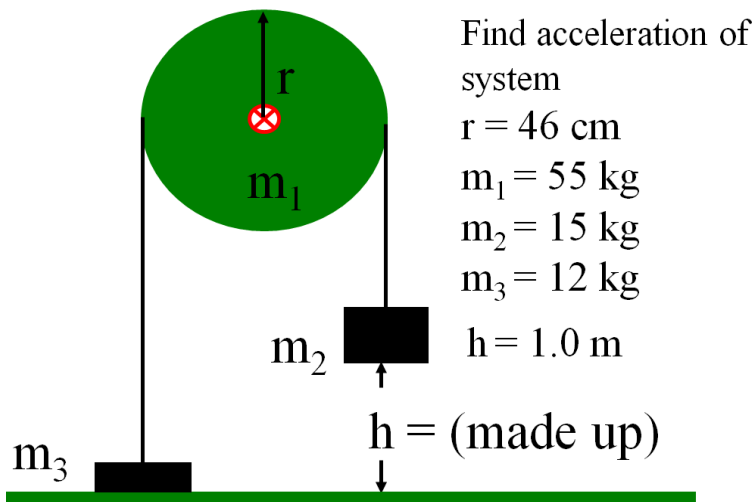
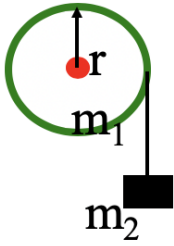
$$r = 12.5 \text{ cm}$$

$$m_1 = 15.7 \text{ kg}$$

$$m_2 = 0.543 \text{ kg}$$

$$h = 0.195 \text{ m}$$

A string is wrapped around a 12.0 cm radius 4.52 kg thin ring. A mass of 0.162 kg is hanging from the end of the string. What is the acceleration of the system, and what is the velocity of  $m_2$  when it has fallen 1.00 m? (Assume it is released from rest)



**Noteguide for Angular Momentum (Videos 8N, 8O, 8P)**

Name \_\_\_\_\_

**8N:  $p = mv, L = I\omega$**

Example: What is the angular momentum of a 23 cm radius 5.43 kg grinding wheel at 1500 RPMs?

Whiteboards:

|  |  |
|--|--|
| 1. What is the Angular Momentum of an object with an angular velocity of 12 rad/s, and a moment of inertia of 56 kgm <sup>2</sup> ? (670 kg m <sup>2</sup> /s) | 2. What must be the angular velocity of a flywheel that is a 22.4 kg, 54 cm radius cylinder to have 450 kgm <sup>2</sup> /s of angular momentum? (140 rad/s) |
|--|--|

**8O:  $Ft = m\Delta v, \Gamma t = I\Delta\omega$**

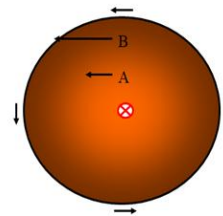
Example: A merry go round that is a 340. kg cylinder with a radius of 2.20 m. If a torque of 94.0 mN acts for 15.0 s, what is the change in angular velocity of the merry go round?

Whiteboards:

|  |  |
|--|--|
| 1. For what time does a torque of 12.0 mN need to be applied to a cylinder with a moment of inertia of 1.40 kgm <sup>2</sup> so that its angular velocity increases by 145 rad/s? (16.9 s) | 2. A grinding wheel that is a 5.60 kg 0.125 m radius cylinder goes from 152 rad/s to a halt in 22.0 seconds. What was the frictional torque? (0.302 mN ) |
|--|--|

## Angular Momentum is Conserved just like linear momentum!!!

$$8P: I_1\omega_1 = I_2\omega_2$$



Example: A figure skater spinning at 3.20 rad/s pulls in their arms so that their moment of inertia goes from  $5.80 \text{ kgm}^2$  to  $3.40 \text{ kgm}^2$ . What is their new rate of spin? What were their initial and final kinetic energies?  
(Where does the energy come from?)

Example: A merry go round is a 210 kg 2.56 m radius uniform cylinder. Three 60.0 kg children are initially at the edge, and the MGR is initially moving at 23.0 RPM. What is the resulting angular velocity if they move to within 0.500 m of the center?

### Whiteboards:

1. A gymnast with an angular velocity of 3.4 rad/s and a moment of inertia of  $23.5 \text{ kgm}^2$  tucks their body so that their new moment of inertia is  $12.3 \text{ kgm}^2$ . What is their new angular velocity?  
(6.5 rad/s)

2. A  $5.4 \times 10^{30} \text{ kg}$  star with a radius of  $8.5 \times 10^8 \text{ m}$  and an angular velocity of  $1.2 \times 10^{-9} \text{ rad/s}$  shrinks to a radius of 1350 m. What is its new angular velocity? hint  
(480 rad/s)

3. A 12 kg point mass on a massless stick 42.0 cm long has a tangential velocity of 2.0 m/s. How fast is it going if it moves in to a distance of 2.0 cm? hint  
(2100 rad/s)

## Angular Energy and Momentum problems from 8.3

### A Basic Energy:

1. What is the kinetic energy of a flywheel with a moment of inertia of  $12.4 \text{ kgm}^2$  that is spinning at  $17.8 \text{ rad/s}$ ? (1960 J)
2. A flywheel spins at  $87.0 \text{ rot/s}$  when it is storing  $12,500 \text{ J}$  of kinetic energy. What is its moment of inertia? ( $0.0837 \text{ kgm}^2$ )
3. What is the speed in RPMs of a  $4.50 \text{ kg}$   $34.0 \text{ cm}$  diameter cylindrical grinding disk if it has  $340. \text{ J}$  of rotational kinetic energy? (977 RPM)
4. A  $4.50 \text{ kg}$   $12.0 \text{ cm}$  radius bowling ball is rolling at  $3.20 \text{ m/s}$ . What is its translational kinetic energy? What is its rotational kinetic energy? What is its total kinetic energy? If it rolled from rest down a hill, how high is the hill? ( $23.0 \text{ J}$ ,  $9.22 \text{ J}$ ,  $32.3 \text{ J}$   $0.731 \text{ m}$ )
5. If linear work is given by  $W = Fs$ , then angular work is  $W = I\theta$ . Use energy to find the angular final velocity of a flywheel that has a moment of inertia of  $8.50 \text{ kgm}^2$  after it has been sped from rest up by a torque of  $52.0 \text{ mN}$  through  $84.0$  radians. ( $32.1 \text{ Rad/s}$ )

### B Rolling problems:

For all of these:

- a. Set up the appropriate dynamics or conservation of energy equation, substitute for  $\omega$  or  $\alpha$ , and for  $I$ , and solve for  $v$  or  $a$ . Show your steps Give an exact answer. (you will need to give an answer with a simplified fraction!)
  - b. Solve for the final velocity of the marble at the bottom of the incline.
  - c. Calculate the acceleration of the marble as it rolls down the incline.
1. A  $11.0 \text{ g}$ ,  $0.0110 \text{ m}$  radius unique circular solid with a moment of inertia given by  $\frac{2}{3}mr^2$ , rolls down an incline that is  $2.60 \text{ m}$  long, and loses  $0.560 \text{ m}$  of elevation. ( $2.80 \text{ m/s}$ ,  $1.51 \text{ m/s/s}$ )
  2. A  $13.0 \text{ g}$ ,  $0.0130 \text{ m}$  radius unique circular solid with a moment of inertia given by  $\frac{1}{2}mr^2$ , rolls down an incline that is  $5.10 \text{ m}$  long, and loses  $1.90 \text{ m}$  of elevation. ( $4.99 \text{ m/s}$ ,  $2.44 \text{ m/s/s}$ )
  3. A  $15.0 \text{ g}$ ,  $0.0140 \text{ m}$  radius unique circular solid with a moment of inertia given by  $\frac{1}{3}mr^2$ , rolls down an incline that is  $4.10 \text{ m}$  long, and loses  $1.30 \text{ m}$  of elevation. ( $4.37 \text{ m/s}$ ,  $2.33 \text{ m/s/s}$ )
  4. A  $143.0 \text{ g}$ ,  $0.0450 \text{ m}$  radius unique circular solid with a moment of inertia given by  $\frac{2}{7}mr^2$ , rolls down an incline that is  $3.30 \text{ m}$  long, and loses  $1.10 \text{ m}$  of elevation. ( $4.10 \text{ m/s}$ ,  $2.54 \text{ m/s/s}$ )
  5. A  $12.0 \text{ g}$ ,  $0.0120 \text{ m}$  radius unique circular solid with a moment of inertia given by  $\frac{7}{8}mr^2$ , rolls down an incline that is  $3.20 \text{ m}$  long, and loses  $0.340 \text{ m}$  of elevation. ( $1.89 \text{ m/s}$ ,  $0.556 \text{ m/s/s}$ )

### C Basic Momentum:

1. What is the angular momentum of a disk with a moment of inertia of  $0.145 \text{ kgm}^2$  that is spinning at  $45.0 \text{ rad/s}$ ? ( $6.53 \text{ kgm}^2/\text{s}$ )
2. What angular velocity in rad/s must a  $120. \text{ kg}$   $1.80 \text{ m}$  radius cylindrical merry go round go to have  $2360 \text{ kg m}^2/\text{s}$  of angular momentum? ( $12.1 \text{ rad/s}$ )
3. What torque would speed up a merry go round with  $296 \text{ kgm}^2$  of rotational inertia from rest to  $6.28 \text{ rad/s}$  in  $32.0$  seconds? ( $58.1 \text{ mN}$ )
4. A  $2.60 \text{ kg}$  cylindrical flywheel with a diameter of  $54.0 \text{ cm}$  is spinning at  $115 \text{ rad/s}$ . If a frictional torque of  $1.30 \text{ mN}$  acts on it, in what time would it stop? ( $8.38 \text{ s}$ )
5. A ballerina spinning at  $1.20 \text{ rev/sec}$  with a moment of inertia of  $2.60 \text{ Kg m}^2$  pulls her arms in so that her new moment of inertia is  $1.80 \text{ Kg m}^2$ . What is her new angular speed? ( $1.73 \text{ rev/sec}$ )
6. A group of children playing on a merry go round spinning at  $52.0 \text{ rpm}$  with a moment of inertia of  $200. \text{ Kg m}^2$  move to its center so that the new angular velocity is  $86.7 \text{ RPM}$ . What is the new moment of inertia? ( $120. \text{ kgm}^2$ )

### D Momentum Questions:

1. **A  $54.0 \text{ kg}$  child is  $1.80 \text{ m}$  from the center of a  $2.10 \text{ m}$  radius merry go round that is a  $170. \text{ kg}$  cylinder.**
  - a. If a torque of  $92.0 \text{ mN}$  is applied for  $13.0$  seconds, what is the change in angular velocity? b. The child moves out to a distance of  $2.10 \text{ m}$ , and as a result the merry go round is spinning at  $0.450 \text{ rot/s}$ . What was its initial angular velocity in  $\text{rot/s}$ ? ( $2.18 \text{ rad/s}$ ,  $0.502 \text{ rot/s}$ )
2. **A  $68.0 \text{ kg}$  child is  $2.70 \text{ m}$  from the center of a  $3.30 \text{ m}$  radius merry go round that is a  $140. \text{ kg}$  cylinder.**
  - a. For what time must a torque of  $31.0 \text{ mN}$  act to accelerate the merry go round from rest to  $5.20 \text{ rad/s}$ ? b. When the merry go round is spinning at  $21.0 \text{ RPM}$ , the child moves in to a distance of  $1.90 \text{ m}$  from the center. What is the final angular velocity in  $\text{RPM}$ ? ( $211 \text{ s}$ ,  $26.2 \text{ RPM}$ )
3. **A  $51.0 \text{ kg}$  child is  $1.10 \text{ m}$  from the center of a  $2.40 \text{ m}$  radius merry go round that is a  $160. \text{ kg}$  cylinder.**
  - a. If the merry go round speeds up from rest to  $4.70 \text{ rad/s}$  in  $14.0$  seconds, what torque was acting? b. When the merry go round is rotating at  $0.970 \text{ rot/s}$ , the child moves out to a distance of  $2.30 \text{ m}$  from the center. What is the new angular velocity of the merry go round in  $\text{rot/s}$ ? ( $175 \text{ mN}$ ,  $0.694 \text{ rot/s}$ )
4. **A  $41.0 \text{ kg}$  child is  $2.20 \text{ m}$  from the center of a  $2.40 \text{ m}$  radius merry go round that is a  $150. \text{ kg}$  cylinder.**
  - a. If a torque of  $95.0 \text{ mN}$  acts on the merry go round for  $8.00$  seconds, what is the change in angular velocity? b. The child moves in to a distance of  $1.10 \text{ m}$  from the center, and as a result, the angular velocity of the merry go round is  $65.0 \text{ RPM}$ . What was the initial angular velocity in  $\text{RPM}$ ? ( $1.21 \text{ rad/s}$ ,  $49.7 \text{ RPM}$ )
5. **A  $58.0 \text{ kg}$  child is  $1.00 \text{ m}$  from the center of a  $2.20 \text{ m}$  radius merry go round that is a  $180. \text{ kg}$  cylinder.**
  - a. For what time must a torque of  $35.0 \text{ mN}$  act on the merry go round to change its angular velocity from rest to  $3.50 \text{ rad/s}$ ? b. If the merry go round is spinning at  $0.780 \text{ rot/s}$ , and the child moves out to  $2.20 \text{ m}$  from the center, what is the final angular velocity in  $\text{rot/s}$ ? ( $49.4 \text{ s}$ ,  $0.537 \text{ rot/s}$ )

### So you think you're so dang smart?

Giancoli #67: Suppose a  $55\text{-kg}$  person stands at the edge of a  $6.5\text{-m}$  diameter merry-go-round turntable that is mounted on frictionless bearings and has a moment of inertia of  $1700 \text{ kgm}^2$ . The turntable is at rest initially, but when the person begins running at a speed of  $3.8 \text{ m/s}$  (with respect to the turntable) around its edge, the turntable begins to rotate in the opposite direction. Calculate the angular velocity of the turntable.  
( $-0.30 \text{ rad/s}$ )

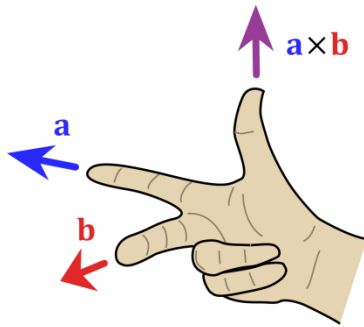




**Vector Cross Product:**

**$A \times B = AB \sin(\theta)$  in the right hand direction**

**The Right Hand Direction is Funky:**



Using your Right hand:

- Index Finger:** First vector (a in this case)
- Middle Finger:** Second Vector (b in this case)
- Thumb:** Direction of the cross product

Note that cross products are NOT commutative. ( $A \times B = -B \times A$ )

Whiteboards: (. is out of the page, and x is into the page. The x in the middle just means cross product)

|                        |                    |             |              |
|------------------------|--------------------|-------------|--------------|
|                        |                    |             |              |
| <b>Out of the page</b> | <b>Up the page</b> | <b>Left</b> | <b>Right</b> |

So Gyroscopes precess because of torque:

**$\Gamma = r \times F$**

A wheel spinning anti clockwise has an angular velocity and momentum that is represented by a vector pointing straight at you. (This is another right hand rule that I will explain in class) The tip of that angular momentum vector will go in the direction of  $r \times F$  using the right hand rule above.

Watch the Ve video and we will do some examples in class.



## Gyroscope Investigation

1. You will need a gyroscope, a gear puller, a gyroscope stand, and a love for rotational mechanics.
2. Get the gyroscope spinning by using the gear puller. Hold the gyroscope firmly, and pull the handle - being careful not to strip the little teeth. Play with it over a table. If it drops on the floor it will break. Come up with a stupid gyroscope trick.
3. Get the gyroscope spinning anti-clockwise as seen from above (This way the L vector is pointing up), put the bottom of the gyro into stand. Note carefully which way the gyroscope precesses.
4. **Draw careful diagrams** that a) show the direction of the torque on the gyroscope (Due to gravity) the axis about which this torque acts is the stand  $\Gamma = r \times F$ , so r is away from the stand, F is straight down, b) show the direction of the angular momentum vector, c) show that the direction of precession has the tip of the L vector going in the direction of the torque.
5. Answer these questions:
  - A. Measure the mass of the rotor by weighing the gyroscope, and subtracting the 23.5 grams that is the cage. Measure the radius of the rotor, and use a formula like  $I = mr^2$  (or is it  $.9mr^2$ ? - feel free to make your own formula) to **find the moment of inertia of the rotor in  $\text{kgm}^2$** .
  - B. The axle has a diameter of 3.9 mm (0.0039 m). Supposing the puller was moving about 1.5 m/s at the end of your pull,
    1. **calculate the angular velocity of the gyro**, (use  $v = \omega r$ ) and
    2. **calculate the angular momentum of the gyro**. ( $L = I\omega$ )
  - C. Calculate the **angular momentum of the earth** ( $L = I\omega$ ). (use the interwebs to find the mass, radius, and period of rotation. Assume the earth is a sphere ( $\frac{2}{5}mr^2$ ).  $\omega = 2\pi/T$ . You can also just go to wolfram alpha and type "angular velocity of the earth") Show that it is about  $7 \times 10^{33}$   $\text{kgm}^2/\text{s}$  (be a bit more exact)
6. Leave your gyroscope exactly the way you found it.

Here's what you turn in:

1. The diagram as explained in part 4.
2. The answers for part 5.